M.Sc. - Mathematics

I Semester End Examination - May 2022
Topology – I

Course Code: MM103T | **QP Code: 11003**
Time: 3 hours | **Total Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.
2) Answer **any five** full questions.

1. (a)Show that
(i) Superset of an infinite set is infinite. (ii) Subset of a finite set is finite.
(b) Define finite and infinite sets. Let $g: X \to Y$ be an one – one correspondence. If the set X is infinite then prove that Y is infinite.
(7+7)

2. (a) Define denumerable set. Prove that every infinite subset of a denumerable set is denumerable.
(b) State and prove Schroder – Berstein theorem.
(7+7)

3. (a) Define metric space. Suppose (X, d) is a metric space, let $d_1(x, y) = \begin{cases} 0 \; if \; x = y \\ 1 \; if \; x \neq y \end{cases}$ be defined on $X \times X$. Prove that d_1 is a metric on X.
(b) Prove that a subspace Y of a complete metric space (X, d) is complete if and only if Y is closed.
(7+7)

4. (a) State and prove contraction mapping theorem.
(b). If (X, d) is totally bounded then prove that it is bounded but not conversely true. (8+6)

5. (a) For any $A \subseteq (X, \tau)$ prove that $A \cup d(A)$ is closed set.
(b) Prove that a set A is closed if and only if $d(A) \subseteq A$.
(c) For any set A in (X, τ) (i) A is open $\Leftrightarrow A = A^0$ (ii) $A \subseteq B$ then $A^0 \subseteq B^0$.
(5+5+4)

6. (a) Let (X, τ) be a topological space. Show that a sub family $\mathfrak{B}$ of τ is a base for τ if and only if for every $U \in \tau$ and $x \in U$ there is a $B \in \mathfrak{B}$ such that $x \in B \in U$.
(b) Prove that a function $f: X \to Y$ is continuous if and only if inverse of open set is open.
(7+7)

7. (a) Let (X, τ) be a topological space. Prove the following
(i) X is neighbourhood of every point.
(ii) If A is a neighbourhood of x and $A \subseteq B$ then B is also a neighbourhood of x.
(b) Show that a bijective function $f: X \to Y$ is homeomorphism if and only if $f(\bar{A}) = \overline{f(A)}$ for all $A \subseteq X$
(7+7)

8. (a) Show that closure of a connected set is connected.
(b) Prove that union of family of connected sets with non-empty intersection is connected.
(c) Give an example to show that a connected space is not locally connected.
(5+5+4)